

# OPERATIONS RESEARCH

[For students of Engineering, Agricultural Sciences, Commerce,  
BBA, MBA, CA & ICWA]

**DR. P.K. MOHANTY**

*Ph.D., F.I.E.*

*Director, Flood Control & Drainage,  
Deptt of Water Resources, Govt. of Odisha,  
BHUBANESWAR.*

**DR. S.K. PATEL**

*Ph.D., F.I.E.*

*Associate Professor,  
Deptt of Mechanical Engineering,  
National Institute of Technology,  
ROURKELA.*



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**THIS BOOK IS DEDICATED  
AT THE LOTUS FEET OF**



**Late GOURISHYAM MOHANTY**



**Late PANCHANAN PATEL**



# FOREWORD

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In the present world, Operations Research is finding applications in almost all business activities. It is next to impossible to complete education in engineering or management studies, without the subject, Operations Research / Quantitative Techniques. It has rather become indispensable in all phases of human activities. Keeping these in view, the endeavor of the authors in bringing out this text book meant for students as well as business managers deserves commendations.

I have found four basic goals those have been achieved by the book- a strong real world focus, structured approach to the subject matter, lucidity of style and simplicity of expression of many complex mathematical formulae and intuitiveness. Good number of solved examples of various types given in each chapter would help the students in better understanding of the techniques. Further, the students preparing for various competitive examinations would find the objective type questions quite useful.

I am sure the book is a treasure for the students, researchers, academicians and users of academicians as well as business communities.

**Prof. Bijoy Kumar Nanda**

Professor, Department of Mechanical Engineering,  
National Institute of Technology, Rourkela  
Former Vice Chancellor,  
Veer Surendra Sai University of Technology, Burla



# PREFACE

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Now-a-days, many business activities suffer setbacks due to lack of intelligent application of appropriate tools in decision-making while facing unwieldy and complex problems. As decision making is the fundamental part of the management process, unscientific and unmethodical approach has made many managers incompetent. This has necessitated in adopting and applying quantitative techniques to aid the process of decision-making in an ever-increasing measure, notwithstanding the techniques being expected only to supplement the manager's sense of decision making and not supplant him. An able manager fully understands the underlying assumptions like, what the analysis achieves, what compromises the decision model makes with reality, and above all, how the conclusions derived are to be adapted to the dynamic business scenarios. Undoubtedly, knowledge of quantitative analysis is a boon to the business manager. Besides, the subject matter has become an integral constituent of the management curricula of nearly all universities and professional institutions, such as the Institute of Chartered Accountants of India, the Institute of Cost and Works Accountants of India, under-graduate studies in construction management and engineering sciences etc.

The present book is intended to provide a comprehensive presentation of the essential topics of quantitative decision-making in a lucid way. An attempt has been made to cover commonly used quantitative techniques in a wide spectrum of decision-making situations. Intense care has been taken to scrupulously avoid recondite notations and obscure manipulations. Every effort has been made to discuss various topics without resorting to rigorous mathematical intricacy and at the same time without any undue sacrifice in pedagogical foundation. The book can be used as a Bible for the managers of today and tomorrow – the students pursuing post-graduate courses in commerce, management and engineering, the professional courses like CA and ICWA etc. A number of problems from various examinations like CES, IAS etc. have also been incorporated.

The book is divided into fourteen chapters. Chapter-I is introduction to the subject matter with historical developments. Chapter-II deals in *Graphical Method* to give the reader an intuitive idea of what Linear Programming is supposed to achieve. Chapter-III develops *Simplex Method* procedure for solving linear programming problems. The concept of *Duality* is introduced in Chapter-IV along with thorough *Post-Optimality Analysis*. The special class *Transportation Problem* and *Assignment Problem* are discussed in Chapter- V and VI respectively, while Chapter-VII is specially meant for the *Sequencing Problems*. Decision-making rules both under uncertainty and under risk have been discussed in Chapter-VIII on *Decision Theory*, whereas *Game Theory* has been discussed in Chapter-IX, where linear programming to Games and Strategies have also been explained. Chapter-X on *Simulation* provides the application of Monte-Carlo Simulation in a variety of situations. Chapter-XI presents an introduction to *Queuing*

*Theory* where both Poisson and non-Poisson systems are discussed. Chapter-XII offers a brief introduction to the techniques of Network Scheduling by *PERT / CPM*. Chapter-XIII presents a variety of deterministic and probabilistic *Inventory Control* Problems both with and without price-breaks. Lastly, *Integer Programming* is discussed in Chapter-XIV, where Gomory's all-integer method and Branch-and-Bound Technique are discussed in detail.

At the beginning of each section, an *Overview* of the chapter is given, which glances at what the reader can expect from it. Similarly, at the end of each section, important bulleted points are given to remember. Graded sample problems and unsolved problems, mostly drawn from latest examination papers, have been given. These help the students to grasp the theory better and prepare him/her for the next section. *Review Questions*, *Multiple Choice Questions* and *Numerical Problems* with variety have been given with their answers only. To enhance further reading, carefully selected *References* are provided at the end of the book. It is hoped that the book shall find a place in the hearts of the readers. However, there is always scope for further improvement. So, readers' views, constructive criticisms, observations and suggestions will be gratefully received and highly appreciated.

Author P.K.Mohanty wishes to record his great sense of genuflection to elder sister Smt. Sanjukta Pattnaik, who taught him mathematics at elementary level, which had put the foundation in shaping him up to the stage what he is now. Bebi Dei- My knee-bent veneration to you.

Authors gratefully acknowledge the inspirations and encouragements received from Sri Bidhu Bhusan Mishra, Principal, Institute of Entrepreneurship Development (IED), Department of MSME, Govt of Odisha and Dr. Dwitikrishna Panigrahi of Deptt of Agriculture, Govt of Odisha, an eminent scholar and scientist, who stood as guiding beacons during this uphill task.

Vijaya Dashami  
Date: 11.11.2016

**Dr. P.K. Mohanty**  
**Dr. S.K. Patel**



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