

*Introduction
to the*
**Theory of
Boehmians**

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Introduction to the Theory of Boehmians

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To My beloved Granddaughter

Ishita

Preface

Despite including our own research in the theory of Boehmians and Bohemian spaces and consulting and reviewing work of other researchers, it did not appear an easy task to write a moderate text in a very comprehensible manner. This monograph is an outcome of very many scattered research papers which are organized for a systematic reading. For comprehension and accessibility, collection of literature is made warily. In the preparation of the complete text, care is taken to simplify mathematics that is involved. The prerequisites for reading this book are real and complex analysis and to some extent, the functional analysis. We, at this juncture, may not prognosis the quantum of readers of this book, because we, in India (and for that matter in many parts of the world), do not run any course on Bohemian spaces. Moreover, this book is not a university text book. The aim of writing this book is to assist young researchers who plan their research in this direction.

I am grateful to many authors whose work have been freely consulted. I am thankful to Prem Lata Paliwal for the excellent job of typesetting the book and to Pukhraj Paliwal for Proof reading. I am thankful to the Department of Science and Technology, Government of India, New Delhi, India, for sanctioning (Sanction No. HR/UR/49/2010) the DST-USERS (Utilization of Scientific Expertise of Retired Scientist). Last but not the least, I am grateful to Pawan Kumar Sharma, Scientific Publishers (India) for granting to publish this book.

P.K. Banerji

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